Asymptotes

Horizontal
Look at the highest degree exponent of the numerator and the denominator.
If the exponent on top (numerator) is larger than the exponent on the bottom (denominator), then there is no horizontal asymptote.
If the exponent on top is smaller than the exponent on the bottom, then there is a horizontal asymptote at y = 0.

If the exponents are equal, there is a horizontal asymptote at $y = \frac{a}{b}$ where a is the coefficient in front of the numerator's highest degree exponent and b is the coefficient in front of the denominator's highest degree exponent.

Just remember BOBO BOTN EATSDC.

BOBO – Bigger on Bottom, horizontal asymptote at y = 0
BOTN – Bigger on Top, NO horizontal asymptote.
EATSDC – Exponents Are the Same, Divide Coefficients to get the horizontal asymptote.

Example 1: y = $\frac{3x^3+2}{x+4}$ would have no horizontal asymptote since the top has degree 3 and the bottom has degree 1. (Top exponent is larger.)

Example 2: y = $\frac{3x^4+2}{x^6+4}$ would have a horizontal asymptote at y = 0 since the top has degree 4 and the bottom has degree 6. (Bottom exponent is larger.)

Example 3: y = $\frac{3x^3+2}{5x^3+4}$ would have a horizontal asymptote at y = $\frac{3}{5}$ since the top has degree 3 and the bottom has degree 3.

Vertical
The vertical asymptote is determined by what cannot happen in the denominator of the fraction. If you have y = $\frac{5x^3+4x+1}{x-6}$, x cannot equal 6; thus, there is a vertical asymptote located at the line x = 6. If you have y = $\frac{5x^3+4x+1}{2x+1}$, you would have a vertical asymptote at the line x = $-\frac{1}{2}$ since when you solve 2x+1 = 0, you get x = $-\frac{1}{2}$.

Slant

A slant asymptote occurs when the degree of the numerator is one more than the denominator. To find the slant, you must divide the denominator into the numerator to find the equation of the slant asymptote (ignore any remainder part).

If you had y = $\frac{x^2+4x+1}{x+1}$, you would do $x+1\overline{)x^2+4x+1}$, which gives you, excluding the remainder, x + 3. Thus, there is a slant asymptote at y = x + 3.

Hole in the graph

When you have an equation like $\frac{x^2+7x+12}{x+4}$, you notice that the top is one more than the bottom, so you think there will be a slant asymptote. However, if you notice, the expression $\frac{x^2+7x+12}{x+4}$ simplifies to $\frac{(x+4)(x+3)}{x+4}$ which simplifies to just (x + 3). Hence, the equation $\frac{x^2+7x+12}{x+4}$ is nothing more than the graph of y = x + 3. The only problem you must remember is that x ≠ -4 since in the denominator you had x + 4, and we can never give x a value where the denominator turns out to be 0. Thus, there will be a hole at x = -4 on the line y = x + 3.